IMPLEMENTATION OF STIFF VIRTUAL WALLS IN FORCE-REFLECTING INTERFACES

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1. Introduction

In recent years haptic interfaces (also know as manipulanda and hand controllers) have been developed for an impressive array of applications. For instance, Mussa-Ivaldi et al. [18] describe a two degree of freedom manipulandum for studies of multijoint human limb movement, Adelstein and Rosen [1] a two degree-of-freedom manipulandum for studies of involuntary tremor, and Jones and Hunter [14] a pair of one degree of freedom manipulanda for psychophysics studies. Bejczy and Salisbury [2] introduce a six degree-of-freedom hand controller for use in space telerobotics, while Jacobsen, et al. [11] have developed a 22 dof force-reflecting exoskeleton for use in underwater telerobotics. Virtual reality has also provided significant impetus for haptic interface development, as the molecular docking work of Brooks, et al. [4], the virtual musical instruments developed by Cadoz and coworkers [6], and the virtual sandpaper system developed by Minsky [17] attest. Other interesting interface devices include those developed by Russo and Tadros [19], Kazerooni [15], Millman and Colgate [16], Iwata [10], Burdea et al. [5], and Howe [9].

A haptic interface may be thought of as a device that generates mechanical *impedances*. "Impedance," here, should be understood to represent a dynamic (history-dependent) relationship between velocity and force. For instance, if the haptic interface is intended to represent manipulation of a point mass, it must exert on the user's hand a force proportional to acceleration; whereas if it is to represent squeezing of a spring, it must generate a force proportional to displacement.

The performance of a haptic interface is often reported in terms of the *dynamic range* of impedances it may represent. At the low end, the range is typically limited by inherent dynamics of the interface device, such as inertia and friction. Impedance fidelity (i.e., the closeness of the impedance as implemented to the desired impedance) is also affected by factors such as actuator nonlinearity and sensor resolution. At the high end, the range is typically limited by system stability (although it will be argued here that passivity is a better measure), which is affected by a variety of factors including sampling rate, sensor resolution, and actuator/sensor collocation.

In a number of the applications alluded to above, the principal limitation has proven to be the achievable upper limit on impedance [6, 17]. Therefore, a benchmark problem of considerable importance is the implementation of a stiff "wall" (for a more complete list of benchmark problems, see Jex [12]). Contacting a wall may be described as the reversible transition from a region of very low impedance to one of very high impedance.

This paper provides a theoretical analysis (supplemented with discussion of experimental and simulation results) of stiff wall implementation. The main result is a criterion for the passivity of a virtual wall in terms of two nondimensional parameters.

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2. Wall Implementation

Wall implementations differ according to hardware and software details. The most common approach, which is the basis for the analyses in this paper, requires a backdriveable manipulandum and discrete time controller. The implementation, shown in Figure 1, is piecewise linear. Here, $x(t)$ is the manipulandum displacement, $F(t)$ is the force generated by the manipulandum, x_{wall} is the location of the virtual wall, K is a virtual stiffness and B is a virtual damping coefficient. Because the manipulandum is backdriveable, the force applied by the operator, $F_o(t)$, is approximately equal to $F(t)$. The velocity sensor has been shown as optional because velocity is often obtained by differentiation of position. The analyses in this paper assume a form of digital differentiation.

Before proceeding, it should be emphasized that the implementation in Figure 1 is by no means unique. For instance, nonlinear impedance characteristics or inertia emulation may be included in the software. Variants involving both hardware and software include the use of digitally supervised analog control [1], and of force sensing in conjunction with a velocity-controlled manipulator [3]. Another interesting variant is the use of hybrid motor/brake actuators [19], which has the possible advantage of ensuring dissipative walls.

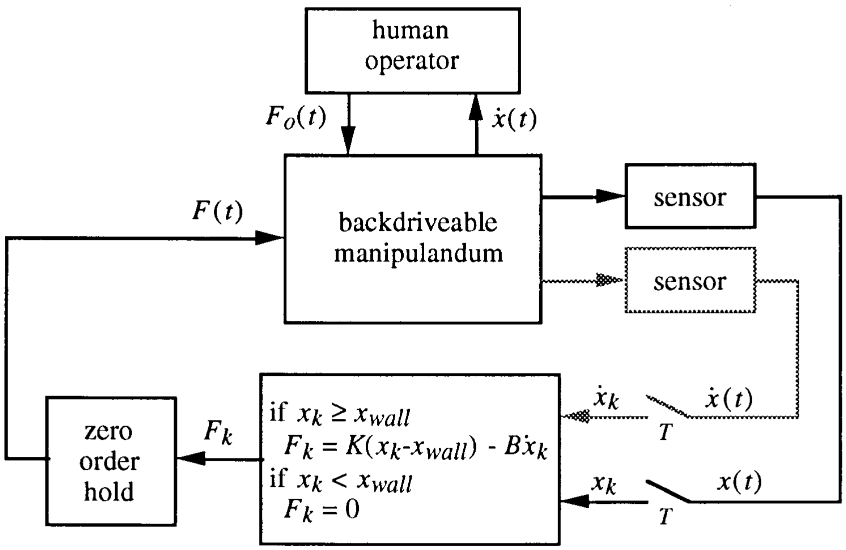

Figure 1. Common Implementation of a Virtual Wall

3. Active Walls

When does a virtual wall feel like a wall, and when does it not? This is a question of psychophysics and is best answered through careful experimentation. There are, however, a number of factors that clearly impact on the answer. For instance, the stiffness (K) must be high. In current generation haptic interfaces, this stiffness is not likely to approach that of a metal on metal or even hard plastic on hard plastic contact. Nonetheless, stiffnesses of 2000-8000 N/m, which are achievable with some designs [9, 13], seem to be sufficient to generate a perception of rigidity. Another factor is damping (B), which must be sufficient to prevent noticeable oscilla-

tions. Unfortunately, increasing B too much tends to cause high frequency oscillations that users often report as a feeling of "rumble."

To date, there has been very little attempt to delineate the regions of parameter space that lead to suitable wall implementations. One exception is a paper by Minsky, et al. [17], which suggests the following relationship between controller gains and sampling rate (T):

$$\frac{B}{KT} > c \tag{1}$$

where c is a constant found to be approximately 0.5. One prediction of this result is that slower update rates can be accommodated by increasing the damping coefficient. The analysis presented here is motivated in part by our inability to reproduce this result. Our experiments suggest that performance is degraded by both increases in B and increases in T.

The criterion above was derived on the basis of wall *stability*. Other parameter constraints (such as the limits on stiffnesses mentioned above) have also been related to stability. Stability, however, is a system property, and therefore dependent upon operator dynamics as well as wall dynamics. An anecdotal observation of ours is that humans are very adept at adjusting their own behavior to destabilize a virtual wall, if possible. In other words, given the opportunity to explore, users will quickly find ways to set up sustained or growing oscillations by gripping the manipulandum lightly or firmly, as necessary. Thus, it appears difficult to use stability as the basis of a performance criterion unless the full range of human dynamics is considered.

A more appropriate basis may be *passivity*. The ability of humans to set up oscillations is evidence of *active walls*: because the frequencies of these oscillations are often outside the range of voluntary motion, and because this behavior is not observed with physical walls, it is evident that the energy supply for the oscillations is the virtual wall, not the human. This motivates the use of a performance criterion based on passivity — the inability to act as an energy source. Passivity is additionally attractive because it is a property of the wall alone.

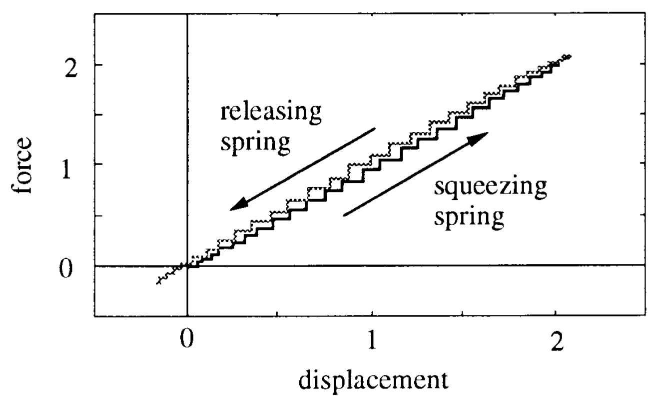

Figure 2. Illustration of energy generation by a virtual spring

Why are virtual walls active? Some insight into this question is provided by analysis of an even simpler system: a virtual spring. An ideal physical spring is a lossless system; therefore, if energy is stored in the spring by squeezing, then removed by releasing, precisely as much energy will be removed as was stored. Now consider the virtual spring. Because it is implemented in discrete time, the force provided by the spring will not increase smoothly with deflection. Instead, the force will be repeatedly "held" at a constant value until updated. Because of this, the average force during squeezing will be slightly less than for a physical spring of identical stiffness, and the average force during release will be slightly greater. This is illustrated in Figure 2. The result is not only that the exquisite balance of stored energy and released energy is lost, but that the spring always acts to store or *generate* energy, never to dissipate energy.

Of course, any real implementation of a virtual wall includes some dissipation. It is straightforward, however, to show that any discrete-time implementation of a virtual dissipator

will be capable of producing energy (usually under high frequency excitation, and wall contact produces high frequencies). One conclusion which may be drawn is that, if a virtual wall is to be passive, it must incorporate some physical dissipation. The amount of inherent dissipation turns out to play an important role in the passivity criterion derived below.

4. Passivity

Consider a one degree of freedom manipulandum. The instantaneous power input to this device is the product of the applied force, $f(t)$, and the velocity, $v(t)$. If the manipulandum is passive, then the integral of power will be greater than or equal to zero for all physically realizable force histories:

$$\int_0^t f(\tau)v(\tau)d\tau \geq 0 \qquad \forall\ f(t),\ t \geq 0 \tag{2}$$

This definition applies equally to linear and nonlinear behaviors, but because it requires examination of *all* force and velocity histories, it is of little practical value. If attention is restricted to linear time-invariant behaviors, then force and velocity are related via an admittance (inverse of impedance) operator, whose transfer function, $Y(s)=V(s)/F(s)$, must satisfy certain easily-tested criteria to be passive [8]. The virtual wall implementation considered here, however, is piecewise linear and is implemented via sampled-data control. As a first step toward a workable theory, the piecewise linear nature of a virtual wall is discarded — only the behavior within the wall will be considered. This step is essential to make the problem linear; its ramifications will be reconsidered in Section 5. Even with this simplification, a novel approach to assessing passivity is needed to cope with the sampled-data aspect of this problem.

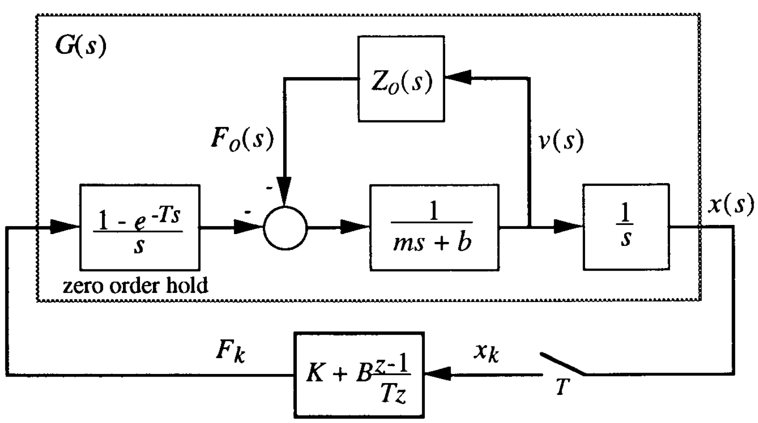

Figure 3. Block diagram of sampled data system for passivity analysis

As a preliminary, consider the following result from "coupled stability" theory [8]: The passivity of a linear time-invariant (LTI) manipulator is a sufficient condition to guarantee stability when the manipulator couples to (contacts) any passive environment, and a necessary condition to guarantee stability if the environment is passive and LTI but otherwise arbitrary. Therefore, proof that a manipulandum with continuous time control is stable when coupled to all LTI passive environments implies that the manipulandum must be passive. In analogy to this result, the approach taken here is to demonstrate that a manipulandum *with sampled-data control* is stable when coupled to all LTI passive environments. Though it has not been proven that this condition is equivalent to Equation 2, it will be taken as a reasonable definition of passivity.

Derivation of the passivity criterion is lengthy and depends on several results that have been presented elsewhere. It will be only outlined here. Figure 3 shows the assumed implementation and manipulator model. Note that the manipulandum possesses inherent dynamics: inertia m, and damping b. Figure 3 also shows that the human operator has been replaced with $Z_o(s)$, which represents the class of *all* LTI, passive impedances. Note finally that the virtual wall is replaced by a discrete time PD controller where the derivative is computed by a backward difference approximation. The stability of this system can be assessed by constructing the Nyquist plot of $G^*(s)H^*(s)$, where:

$$G^*(s) = \frac{1}{T}\sum_{n=-\infty}^{n=\infty} G(s+jn\omega_s) \qquad \omega_s = \frac{2\pi}{T}$$

$$G(s) = \frac{1-e^{-Ts}}{s^2}\frac{1}{ms+b+Z_o(s)} \tag{3}$$

$$H^*(s)\Big|_{s=\frac{1}{T}\ln z} = H(z) \qquad H(z) = K + B\frac{z-1}{Tz}$$

Passivity is implied if this Nyquist plot does not encircle -1 for any choice of $Z_o(s)$.

It can be shown that, at any frequency, the Nyquist plot $1/(ms+b+Z_o(s))$ must lie within a disk of radius $1/2b$, centered at the point $1/2b$+0j. Thus, the fact that $Z_o(s)$ represents a class of impedances rather than a particular impedance produces a simple effect: a point in the Nyquist plane "grows" to a disk of finite radius. Because this disk is frequency-independent, it may be extracted from the summation when computing $G^*(s)$. The term that remains:

$$r(j\omega) = \frac{1}{T}\sum_{n=-\infty}^{n=\infty}\frac{1-e^{-j\omega T}}{(j\omega+j\omega_s n)^2} = (e^{-j\omega T}-1)\,\frac{T}{4}\csc^2\!\left(\frac{\omega T}{2}\right) \tag{4}$$

may be viewed as a frequency-dependent rotation and scaling of this disk. Thus, the set of possible Nyquist plots of $G^*(s)$ may be confined to a well-defined, though frequency dependent region of the complex plane. The method presented in [7] may then be used to find a corresponding region within which the Nyquist plot of $H^*(s)$ (or $H(z)$) must lie to guarantee closed loop stability. This leads to the following result:

$$\left|\frac{r(j\omega)H(e^{j\omega T})}{r(j\omega)H(e^{j\omega T}) + 2b}\right| \le 1 \qquad \forall\ 0 \le \omega \le \frac{\pi}{T} \tag{5}$$

This may be understood as a condition for the passivity of the manipulandum with discrete time controller $H(z)$. Inserting the specified form for $H(z)$ and normalizing, the condition becomes:

$$\left|\frac{(1+\alpha\frac{z-1}{z})\frac{z-1}{z}}{(1+\alpha\frac{z-1}{z})\frac{z-1}{z} - 2\alpha\beta\xi^2}\right| \le 1, \quad z = e^{j\xi}, \quad \forall\ 0 \le \xi \le \pi$$

$$\alpha = \frac{B}{KT} \qquad \beta = \frac{b}{B} \tag{6}$$

It is important to note that this condition depends on only two nondimensional parameters: the ratio of intrinsic to virtual damping, and the ratio of wall time constant (B/K) to sampling period (α is the same parameter identified by Minsky, et al.).

Figure 4 displays a map of the $\alpha\beta$ parameter space, broken into regions that correspond to active and passive behaviors. This graph provides a number of valuable insights. First, it is evident that a certain amount of inherent energy dissipation (b) is essential if a wall is to be passive. Increasing b and decreasing T both improve "passivity margin." Perhaps most importantly, the role of virtual damping is clarified. Increasing B has the beneficial effect of increasing α, but the much stronger detrimental effect of decreasing β. The success that Minsky, et al. had in increasing virtual damping is possibly a result of greater friction in their manipulandum.

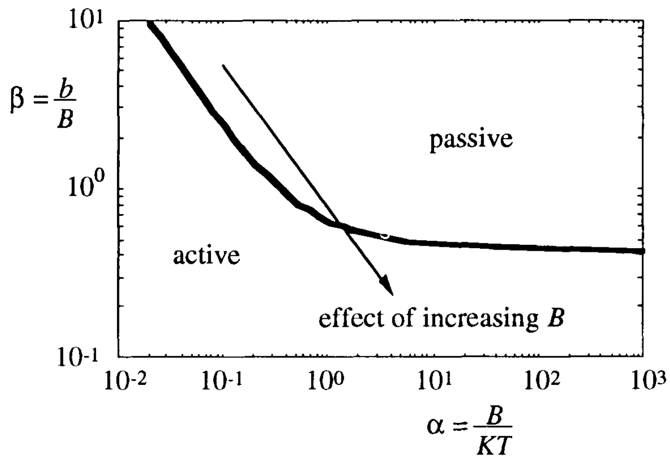

Figure 4. Regions of passive and active behavior in terms of nondimensional parameters.

5. Effect of Piecewise Continuity; Conclusions

In continuous time systems, coupled stability implies contact stability. In other words, if a manipulator with a continuous time controller is stable both in isolation and when fixed to a particular environment, it is also stable when allowed to contact the environment — the act of making and breaking contact cannot be the source of energy. With discrete time control, however, this is not true, and a specific counterexample has been found. Space limitations prevents a detailed elaboration, but in this counterexample, a spring (representing a human arm) is connected to a manipulandum, and the manipulandum is brought into contact with a virtual wall. For the parameters chosen, the system is stable in both the region outside the wall and the region inside the wall, but growing oscillations are observed when the manipulandum bounces against the wall. It is a perplexing result, though clearly related to the fact that, due to discrete time control, the wall does not "turn on" and "turn off" at precisely the same location. It is interesting to note that, in this example, the virtual wall is not passive (it is, however, stable in isolation and when connected to the spring). To date, no example of contact instability with a passive wall has been found. It should also be noted that this effect appears to be experimentally reproducable, when the operator holds the manipulandum against the virtual wall very lightly. While clearly beyond the scope of the theory presented here, this effect merits closer investigation in the future.

The technique presented here can be used to study other wall implementations, as well as the effects of filtering, delay, etc. This work is underway. We are also in the process of developing a one degree of freedom manipulandum that will enable a thorough study of stiff wall implementation and perceptual psychophysics. This device features a high resolution encoder (800,000 counts/rev), torque sensing, and variable intrinsic damping.

Acknowledgments

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